Application of the Direct Quadrature Method of Moments to a Hollow-Cone Water Spray

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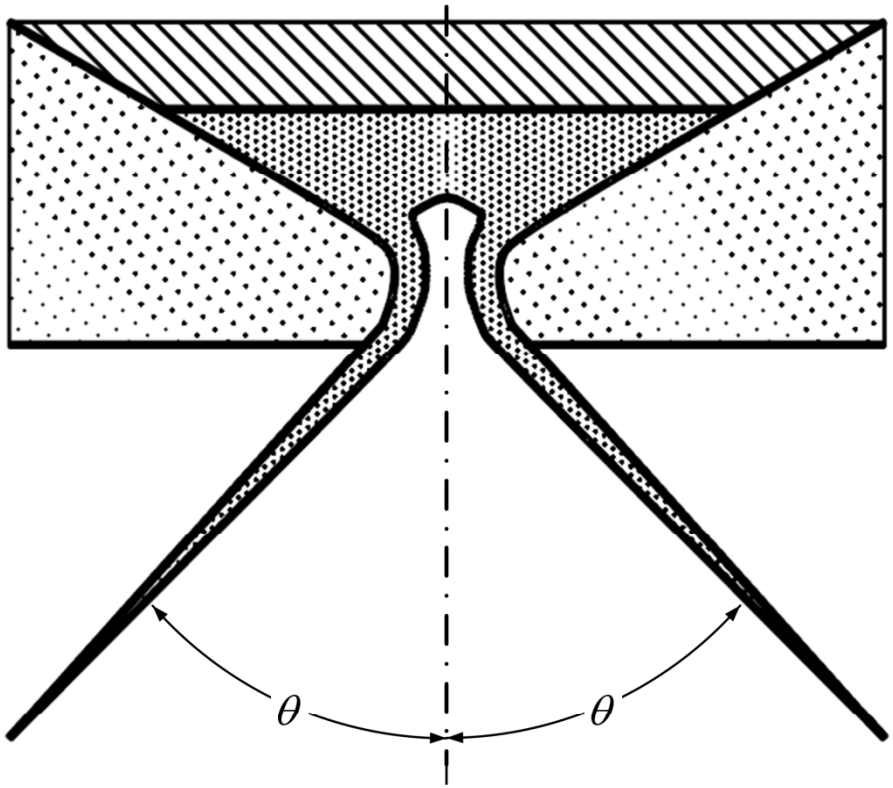
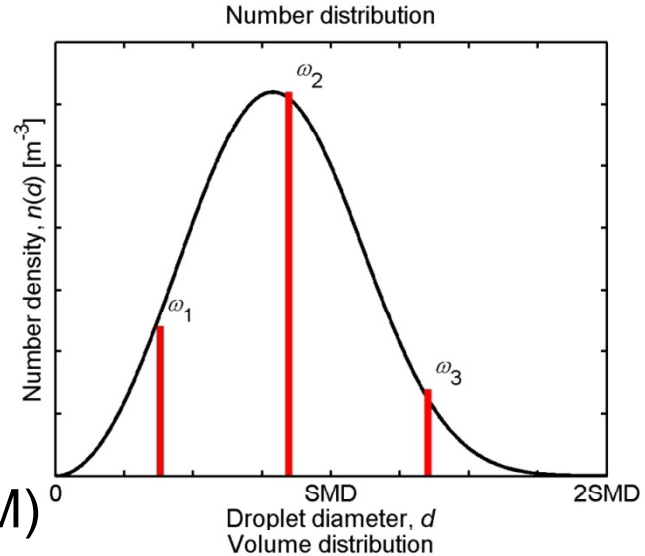
Danfoss Pressure-Swirl Atomizer

- Applications
  - Domestic heating burners
- Daily production
  - About 30 000 nozzles
- Characteristics
  - Air-cored vortex
  - Hollow conical sheet
  - Hollow-cone spray
  - Low capacity
    - 1.46 – 6.55 kg/h
  - Spray angles
    - $2\theta = 30° – 90°$



DQMOM-Multi-Fluid Model

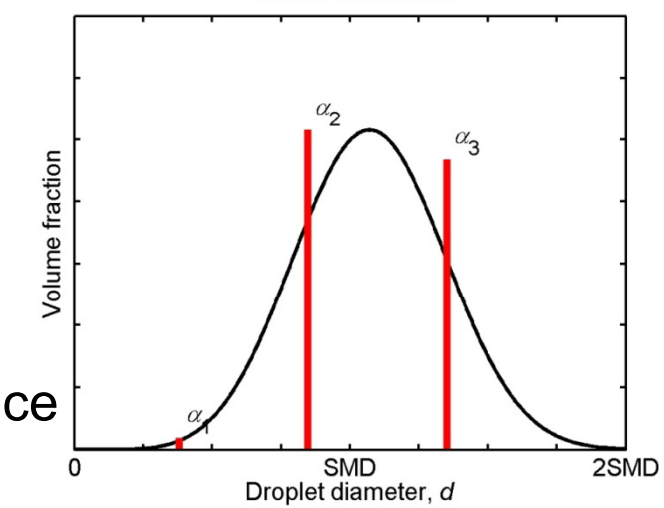
- Eulerian multi-fluid model in Fluent 6.2
  - One gas phase
  - *N* distinct droplet phases
    - One phase for each size class
    - Coupled to population balance equations



- Direct Quadrature Method of Moments (DQMOM)
  - Droplet size distribution (DSD)

$$n(d) = \sum_{q=1}^{N} \omega_q \delta\left[d - d_q\right]$$

  - Transport equations for weight ω_q and weighted abscissa $\delta_q = \omega_q d_q$
  - DSD evolves due to breakup and coalescence



DQMOM Transport Equations

The DQMOM representation of the DSD involves the solution of

Volume fraction, $\alpha_q, q \in [1, N]$

$$\frac{\partial}{\partial t}\left(\alpha_q \rho_l\right) + \frac{\partial}{\partial x_i}\left(\alpha_q \rho_l U_{i,q}\right) = \frac{\pi}{2}\rho_l d_q^2 S_{\delta_q} - \frac{\pi}{3}\rho_l d_q^3 S_{\omega_q}$$

Diameter, $d_q, q \in [1, N]$

$$\frac{\partial}{\partial t}\left(\alpha_q \rho_l d_q\right) + \frac{\partial}{\partial x_i}\left(\alpha_q \rho_l U_{i,q} d_q\right) = \frac{2\pi}{3}\rho_l d_q^3 S_{\delta_q} - \frac{\pi}{2}\rho_l d_q^4 S_{\omega_q}$$

Source terms S_{ω_q} and S_{δ_q}: From first 2N integer moments

$$(1-k)\sum_{q=1}^{N} d_q^k S_{\omega_q} + k\sum_{q=1}^{N} d_q^{k-1} S_{\delta_q} = \bar{S}_{m_k}; \quad k \in [0, 2N-1]$$

Breakup Kernel

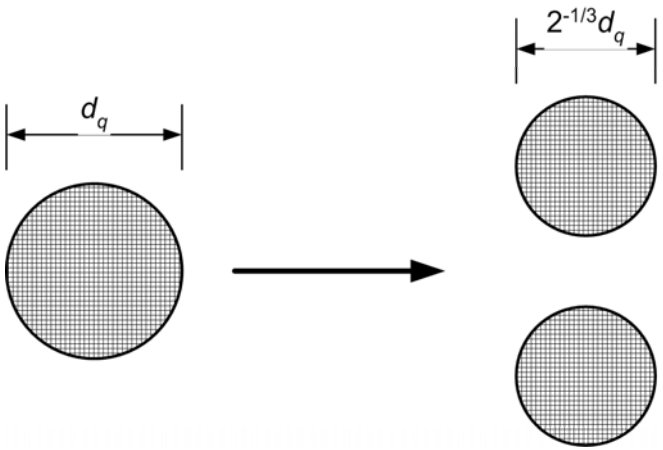
The WAVE atomization model breakup kernel

$$a_q = \frac{1 - d_{st}/d_q}{\left(\bar{b}_q^{(2)}/d_q^2 - 1\right)\tau_{bu}}, \quad d_{st} < d_q$$

where τ_{bu} = breakup time

d_{st} = stable diameter

Breakup daughter distribution function

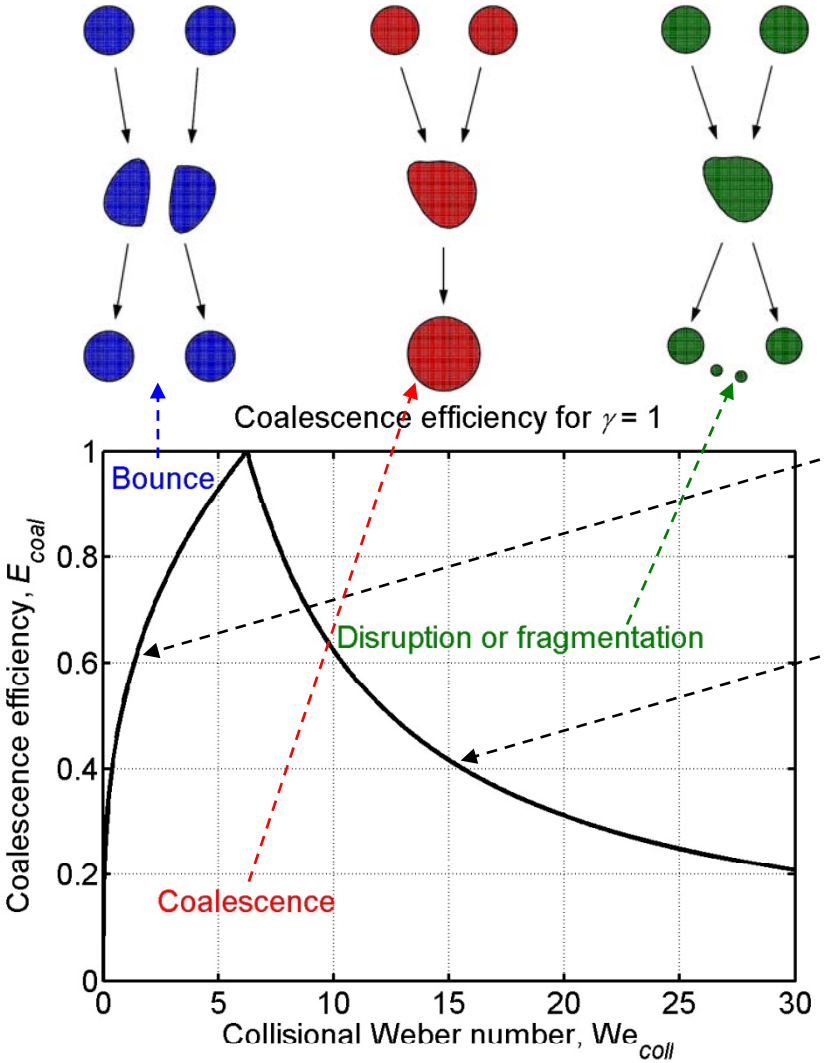


Symmetric breakup:

$$b\left(d \middle| d_q\right) = \begin{cases} 2 & \text{if } d = 2^{-1/3} d_q \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{b}_q^{(k)} = 2^{(3-k)/3} d_q^k$$

Outcome of Collisions



$$We_{coll} = \frac{\rho_l U_{rel}^2 d_{small}}{\sigma}$$

$$E_{boun} = \min\left[1, \left(\frac{We_{coll}}{4.8 f(\gamma)}\right)^{1/3}\right]$$

$$E_{coal} = \min\left[1, \frac{4.8 f(\gamma)}{We_{coll}}\right]$$

$$f(\gamma) = \gamma^3 - 2.4\gamma^2 + 2.7\gamma$$

$$\gamma = \frac{d_{large}}{d_{small}}$$

Collision Kernels

Collision coefficient: $\beta_{pq} = \pi d_{pq}^2 U_{rel}; \quad d_{pq} = \dfrac{d_p + d_q}{2}$

Coalescence kernel: $c_{pq} = \min\left(E_{boun}, E_{coal}\right)\beta_{pq}$

Collision-induced fragmentation kernel: $e_{pq} = \left(1 - E_{coal}\right)\beta_{pq}$

Diameter of droplet fragments (Post and Abraham, 2002):

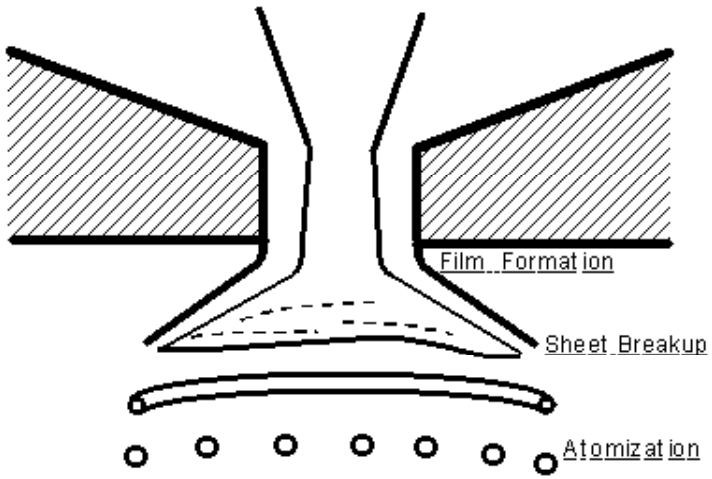
$$d_{frag} = \frac{1.89\left(d_p^3 + d_q^3\right)^{1/3}}{\sqrt{2.81\mathrm{We}_{coll}^{2/7}\left(1+\gamma^3\right)^{2/21} + 1}}$$

LISA Model

Linearized Instability Sheet Atomization (LISA) model (Schmidt *et al.* 1999)

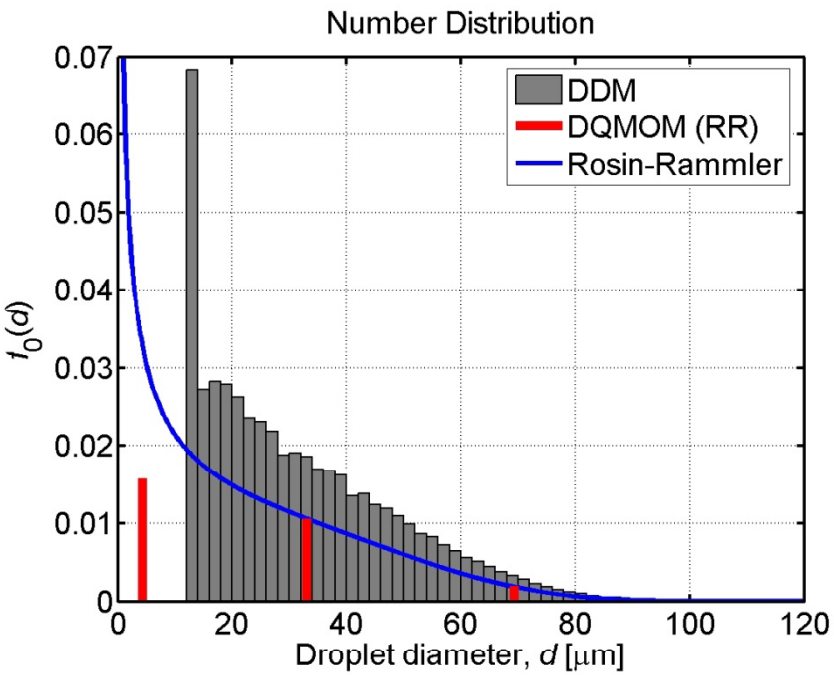
- Film formation
- Sheet breakup and atomization

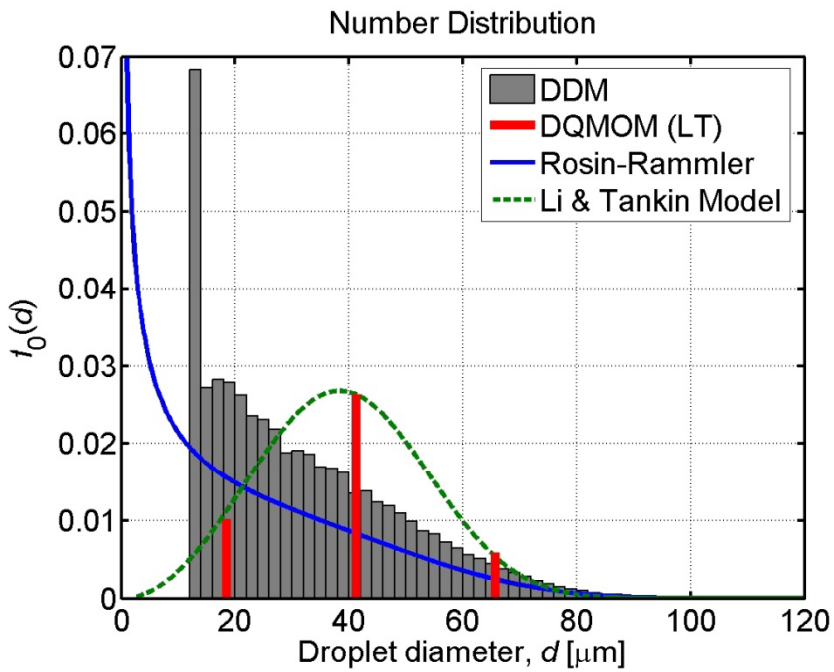
Exit diameter	D_0	436 μm
Injection pressure	ΔP	850 kPa
Mass flow rate	$\dot{m}_l$	3.2 kg/h
Cone angle	2θ	80°
Injection velocity	U_l	28.9 m/s
Film thickness	δ_0	31.6 μm
Sheet thickness parameter	K	3809 μm^2
Sheet breakup time	τ_{bu}	227 μs
Sheet wave length	Λ_S	884 μm
Droplet diameter	d_D	48.6 μm



Initial Droplet Size Distribution

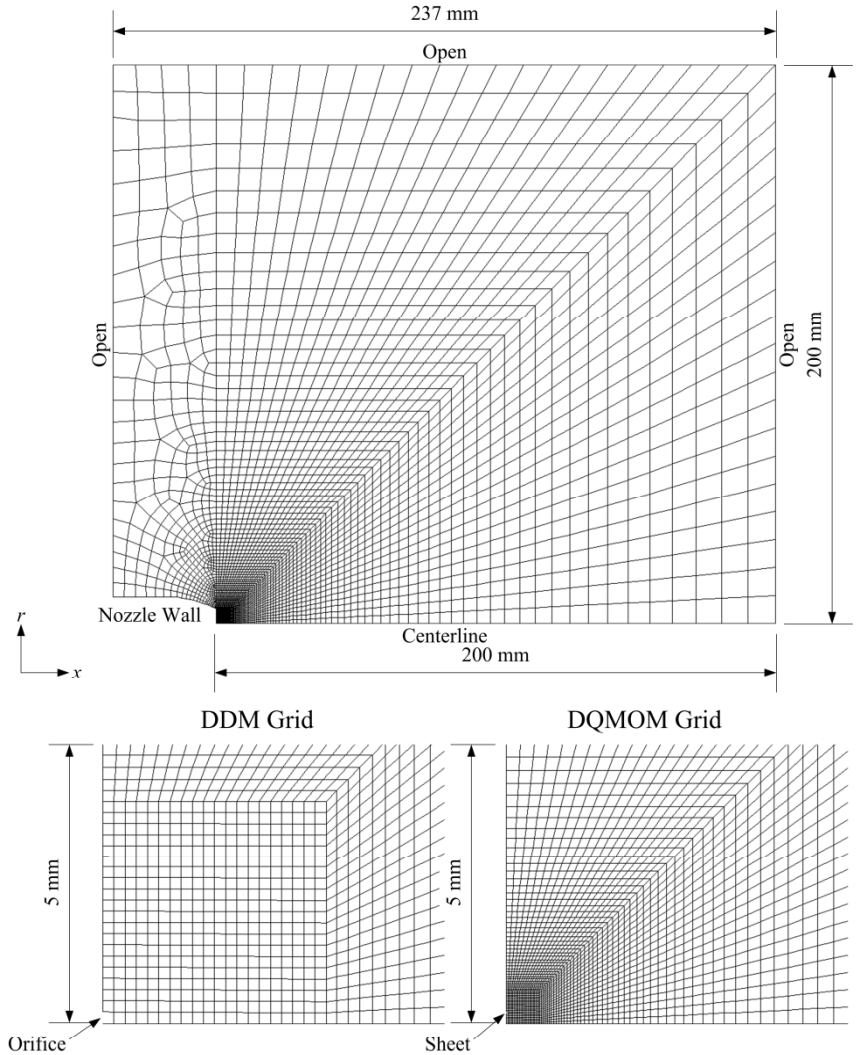
- DDM
  - Selected randomly
  - Rosin-Rammler volume distribution
- DQMOM
  - Rosin-Rammler (RR)
  - Li & Tankin (LT) model
  - Nodes are calculated from 2*N* moments using the PD algorithm





Computational Grid

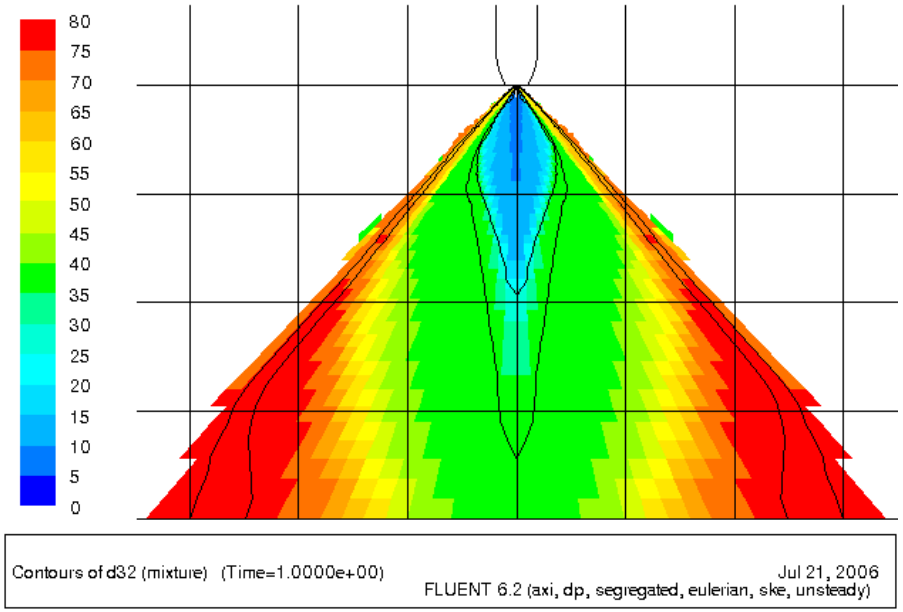
- Domain (r, x > 4 mm)
  - Axisymmetric
  - Unstructured
  - Fine resolution close to orifice
- DDM Grid
  - Orifice is comprised of one cell
  - Size: 200×200 μm
  - Cells: 3,944
- DQMOM Grid
  - Sheet is fitted into one cell
  - Size: 30×30 μm
  - Cells: 5,424



Sauter Mean Diameter (SMD)

- Evolution of Sauter Mean Diameter (SMD), d_{32} [μm]
- Evolution liquid volume fraction (iso-lines: $\alpha_l = 10^{-6}$ and $\alpha_l = 10^{-5}$)

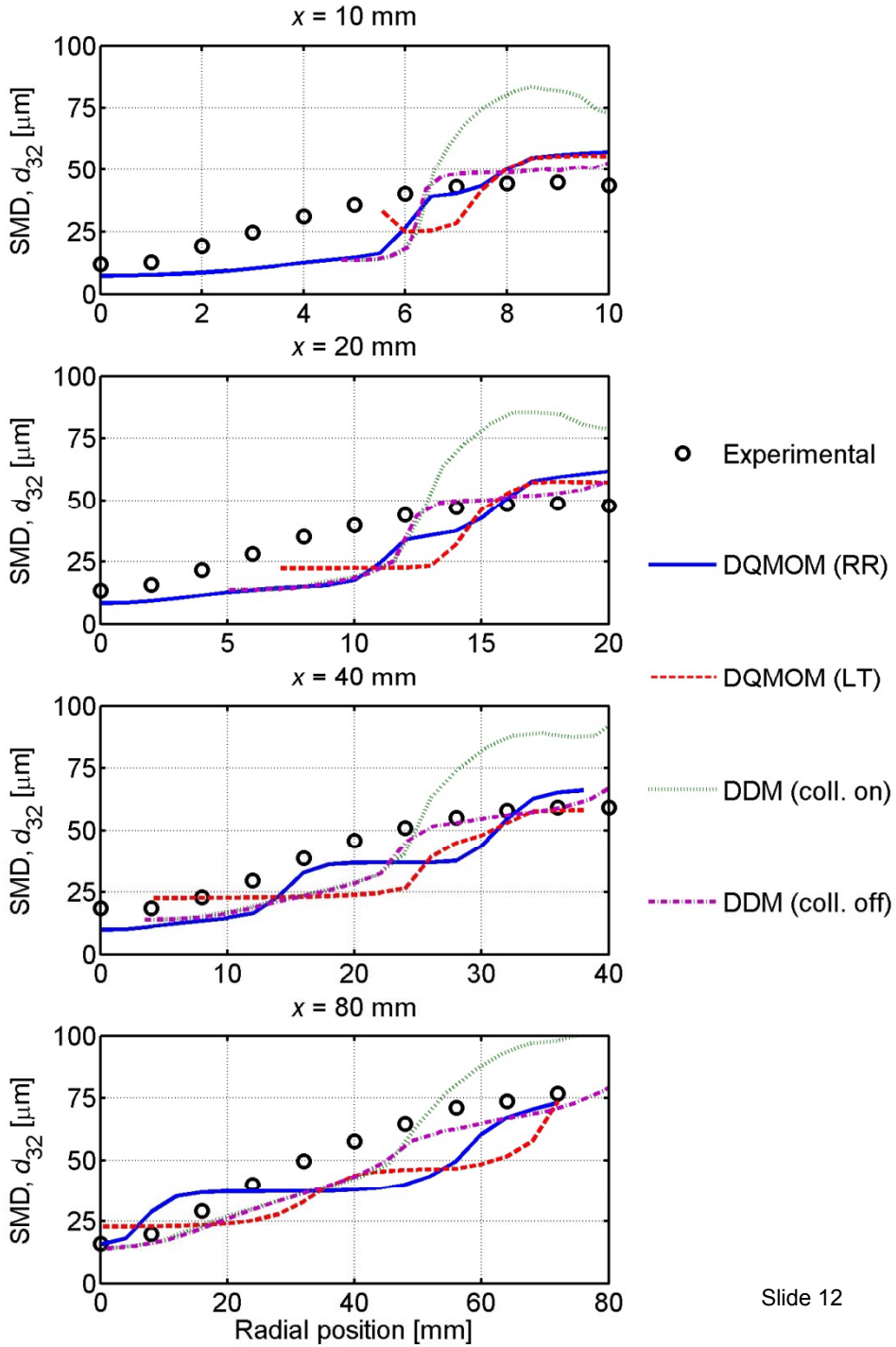
DQMOM (RR)



- The grid superimposed has a spacing of 50×50 mm

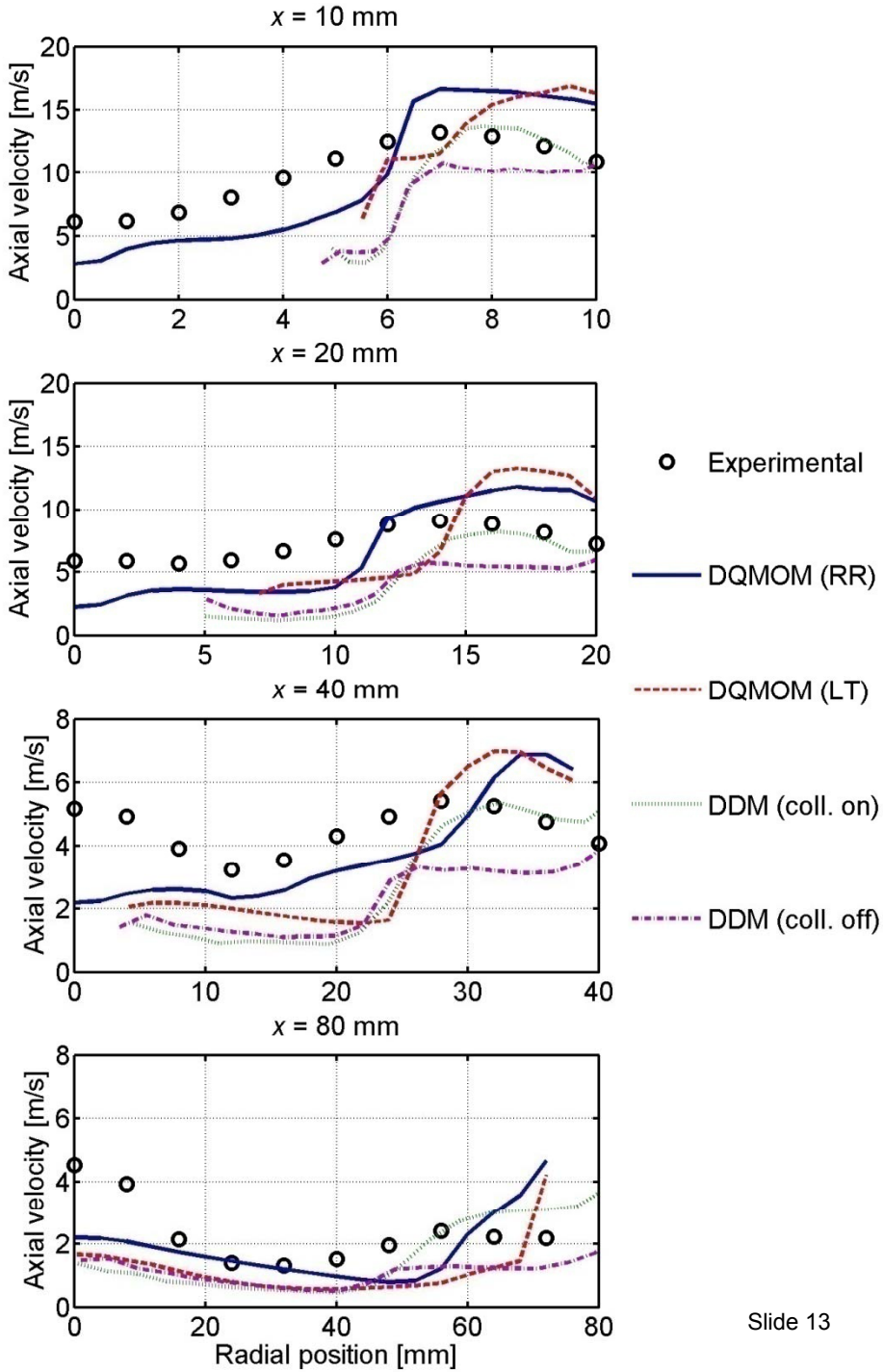
SMD

- Good agreement
- Result of initial DSD
- Effect of DDM collisions model
- DQMOM (RR)
  - Correct trend
  - Slope changes
  - SMD at centerline underpredicted
  - SMD at periphery overpredicted



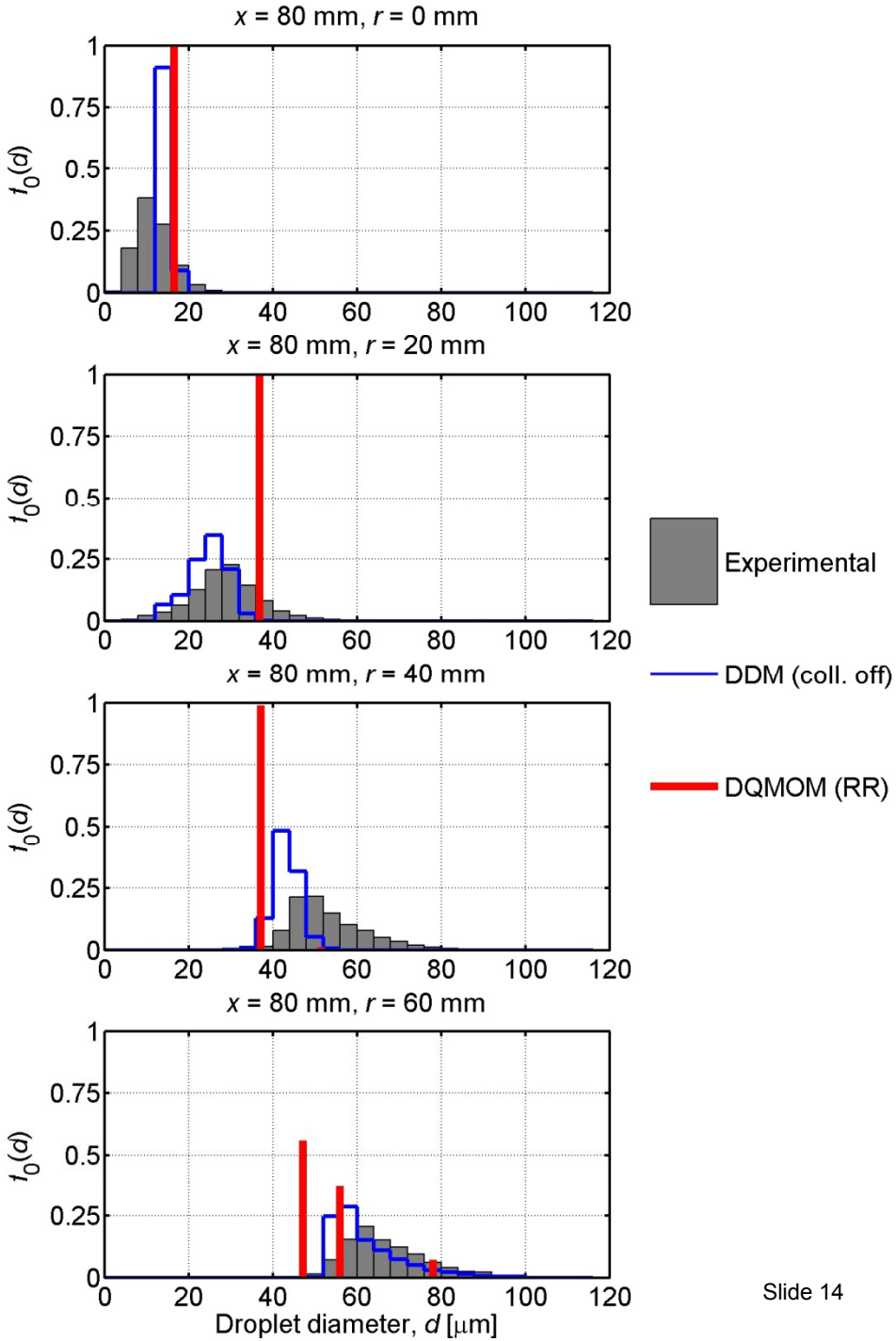
Axial Velocity

- Reasonable accurate
  - Correct trend
- DQMOM (RR) most accurate
  - Centerline values underpredicted
  - Periphery values overpredicted
  - Finer grid resolution compared to DDM



DSD

- Axial location x = 80 mm
- Reasonable good agreement
- Experimental results: wider range
- DQMOM-multi-fluid model
  - Nearly monodisperse DSD's
    - No droplet breakup
    - Few collisions
    - Change due to convection
  - Agreement at shorter distances



Conclusions

- DQMOM-multi-fluid model applied to a low-speed hollow-cone spray
  - Low relative velocities
    - No secondary droplet breakup
  - Wide angle spray
    - Few collisions
  - DSD determined by primary breakup of the liquid sheet
- Predictions compared to
  - Experimental Phase Doppler Anemometry (PDA) data
  - Fluent Discrete Droplet Model (DDM)
- The model for the primary breakup of the liquid sheet is important
  - LISA model appears to be valid
- DQMOM with Rosin-Rammler initial DSD shows reasonable results for
  - Axial droplet velocities
  - Sauter mean droplet diameters
  - Droplet size distributions